Coordenadas Polares

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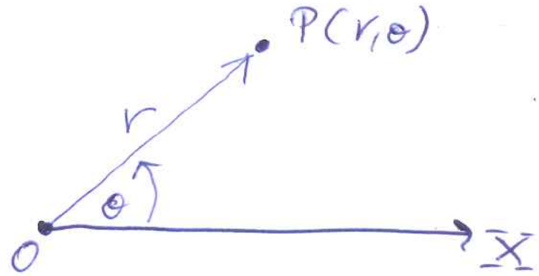
O : polo

$O\underline{X}$: eje polar

r : radio vector

θ : anomalía.

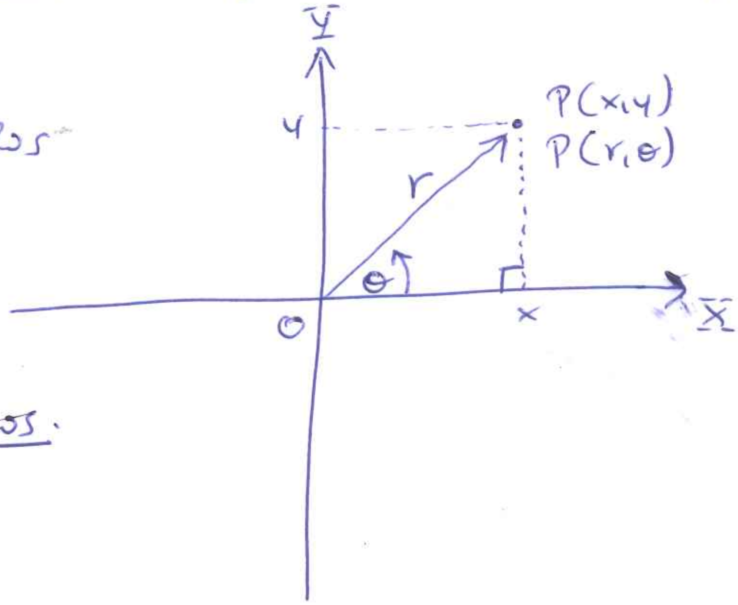


Transformación entre coordenadas

* **De Polares a Cartesianas:**

(r,θ) conocidos $\longrightarrow$ (x,y) desconocidos

$$\cos\theta = \frac{x}{r} \quad , \quad \operatorname{sen}\theta = \frac{y}{r}$$



$$\Longrightarrow \boxed{\begin{array}{l} x(r,\theta) = r\cos\theta \\ y(r,\theta) = r\operatorname{sen}\theta \end{array}}$$

(x,y) únicos.

* **De Cartesianas a Polares:**

$$x^2+y^2 = r^2 \quad , \quad \tan\theta = \frac{y}{x}$$
(Pitágoras)

$$\Longrightarrow \boxed{\begin{array}{l} r(x,y) = \sqrt{x^2+y^2} \\ \theta(x,y) = \operatorname{Arctan}\left(\frac{y}{x}\right) \end{array}}$$

(r,θ) no son únicos:
r puede ser positivo o negativo

$$\theta = \underbrace{\operatorname{Arctan}\left(\frac{y}{x}\right)}_{\text{Valor principal}} \pm k\pi \ , \ k \in \mathbb{Z}$$

① Determinar las coordenadas cartesianas para los puntos siguientes:

Ⓐ $(\sqrt{2}, \pi/4)$ Ⓑ $(-3, 5\pi/6)$ Ⓒ $\left(5, \operatorname{Arctan}\left(\frac{4}{3}\right)\right)$

Sol: Ⓐ $r = \sqrt{2}$, $\theta = \pi/4$

$$x = \sqrt{2}\cos\left(\frac{\pi}{4}\right) = \sqrt{2}\cdot\frac{\sqrt{2}}{2} = \frac{2}{2} = 1$$

$y = \sqrt{2}\ \text{sen}\left(\frac{\pi}{4}\right) = \sqrt{2} \cdot \frac{\sqrt{2}}{2} = \frac{2}{2} = 1.$

$\therefore, \quad (\sqrt{2}, \pi/4)_{Pol} = (1,1)_{Cart.}$

(B) $(-3, 5\pi/6)$

$r = -3 \qquad \theta = 5\pi/6$

$$\begin{aligned} x &= -3\cos(5\pi/6) \\ &= -3 \cdot \cos(\pi/6) \\ &= -3 \cdot \frac{\sqrt{3}}{2} \end{aligned} \qquad\qquad \begin{aligned} y &= -3\ \text{sen}(5\pi/6) \\ &= -3(-\text{sen}(\pi/6)) \\ &= 3 \cdot \frac{1}{2} \end{aligned}$$

$\therefore, \quad (-3, 5\pi/6)_{Pol} = \left(-\frac{3}{2}\sqrt{3}; \frac{3}{2}\right)_{Cart.}$

(C) $\left(5, \text{Arctan}\left(\frac{4}{3}\right)\right)$

$r = 5, \quad \theta = \text{Arctan}\left(\frac{4}{3}\right) \Rightarrow \tan\theta = \frac{4}{3} = \frac{\text{sen}\,\theta}{\cos\theta}$

$\Rightarrow \text{sen}\,\theta = \frac{4}{r}, \quad \cos\theta = \frac{3}{r}$

$\Rightarrow r\,\text{sen}\,\theta = \boxed{4 = y}, \quad r\cos\theta = \boxed{3 = x}$

$\therefore, \quad \left(5, \text{Arctan}\left(\frac{4}{3}\right)\right)_{Pol} = (3,4)$

(2) Determinar las coordenadas polares para los puntos dados:

(A) $(1,1)$ (B) $(-3,0)$ (C) $(\sqrt{3}, -1)$ (Considerar $0 \leq \theta < 2\pi$, $r \geq 0$).

Sol: (A) $(1,1)$ $\quad x = 1,\ y = 1$

$r = \sqrt{(1)^2 + (1)^2} = \boxed{\sqrt{2} = r}$

$\theta = \text{Arctan}\left(\frac{1}{1}\right) = \text{Arctan}(1) = \frac{\pi}{4}$

En general, $\frac{\pi}{4} \pm k\pi$ entre 0 y 2π

$\frac{\pi}{4}$

$\frac{\pi}{4} + \pi = \frac{5\pi}{4}$

· ¿Cómo decidir cuál de los dos sirve? ¡Vea el cuadrante!

* $\frac{\pi}{4}$ está en el 1° cuadrante $\left(0 < \frac{\pi}{4} < \frac{\pi}{2}\right)$

* $\frac{5\pi}{4}$ está en el 3° cuadrante $\left(\pi < \frac{5\pi}{4} < \frac{3\pi}{2}\right)$

* $(1,1)$ está en el 1° cuadrante.

$\therefore$, $(1,1)_{cart} = (\sqrt{2}, \pi/4)_{Pol}$.

Ⓑ $(-3,0)$ $\quad x=-3, \quad y=0$

$r = \sqrt{(-3)^2 + (0)^2} = \sqrt{9} = +3.$

$\theta = \text{Arctan}\left(\frac{0}{-3}\right) = \text{Arctan}(0) = 0 \pm k\pi \begin{cases} 0 \\ \pi \end{cases}$

* $(-3,0)$ está en el eje X negativo $(\theta = \pi)$.

$\therefore$, $(-3,0)_{cart} = (3, \pi)_{Pol}$.

Ⓒ $(\sqrt{3}, -1)$ $\quad x = \sqrt{3}, \quad y=-1$

$r = \sqrt{(\sqrt{3})^2 + (-1)^2} = \sqrt{4} = +2.$

$\theta = \text{Arctan}\left(\frac{-1}{\sqrt{3}}\right) = \text{Arctan}\left(-\frac{\sqrt{3}}{3}\right) = -\frac{\pi}{6} + \pi = \frac{5\pi}{6} \pm k\pi \begin{cases} \frac{5\pi}{6} \\ \frac{11\pi}{6} \end{cases}$

* $(\sqrt{3}, -1)$ está en el 4° cuadrante $\left(\theta = \frac{11\pi}{6}\right)$.

$\therefore$, $(\sqrt{3}, -1)_{cart} = (2, 11\pi/6)_{Pol}$.